Topic 1. Quantitative Methods.
Topic 2. Economics.
Topic 3. Financial Statement Analysis.
Topic 4. Corporate Issuers.
Topic 5. Equity Investments.
Topic 6. Fixed Income.
Topic 7. Derivatives.
Topic 8. Alternative Investments.
Topic 9. Portfolio Management.
Topic 10. Ethical and Professional Standards.

CFA I

Chartered Financial Analyst

FinanCer TRAINING
FINANCIAL CERTIFICATIONS TRAINING

Alexey De La Loma Jiménez
CFA, CMT, CAIA, FRM, EFA, CFTe

Learning Module Overview

- An interest rate, r, can have three interpretations: (1) a required rate of return, (2) a discount rate, or (3) an opportunity cost. An interest rate reflects the relationship between differently dated cash flows.
- An interest rate can be viewed as the sum of the real risk-free interest rate and a set of premiums that compensate lenders for bearing distinct types of risk: an inflation premium, a default risk premium, a liquidity premium, and a maturity premium.
- The nominal risk-free interest rate is approximated as the sum of the real risk-free interest rate and the inflation premium.
- A financial asset’s total return consists of two components: an income yield consisting of cash dividends or interest payments, and a return reflecting the capital gain or loss resulting from changes in the price of the financial asset.
- A holding period return, R, is the return that an investor earns for a single, specified period of time (e.g., one day, one month, five years).
- Multiperiod returns may be calculated across several holding periods using different return measures (e.g., arithmetic mean, geometric mean, harmonic mean, trimmed mean, winsorized mean). Each return computation has special applications for evaluating investments.
- The choice of which of the various alternative measurements of mean to use for a given dataset depends on considerations such as the presence of extreme outliers, outliers that we want to include, whether there is a symmetric distribution, and compounding.
- A money-weighted return reflects the actual return earned on an investment after accounting for the value and timing of cash flows relating to the investment.
- A time-weighted return measures the compound rate of growth of one unit of currency invested in a portfolio during a stated measurement period. Unlike a money-weighted return, a time-weighted return is not sensitive to the timing and amount of cash flows and is the preferred performance measure for evaluating portfolio managers because cash withdrawals or additions to the portfolio are generally outside of the control of the portfolio manager.
- Interest may be paid or received more frequently than annually. The periodic interest rate and the corresponding number of compounding periods (e.g., quarterly, monthly, daily) should be adjusted to compute present and future values.
- Annualizing periodic returns allows investors to compare different investments across different holding periods to better evaluate and compare their relative performance. With the number of compounding periods per year approaching infinity, the interest is compound continuously.

Learning Module Overview

- Gross return, return prior to deduction of managerial and administrative expenses (those expenses not directly related to return generation), is an appropriate measure to evaluate the comparative performance of an asset manager.
- Net return, which is equal to the gross return less managerial and administrative expenses, is a better return measure of what an investor actually earned.
- The after-tax nominal return is computed as the total return minus any allowance for taxes on dividends, interest, and realized gains.
- Real returns are particularly useful in comparing returns across time periods because inflation rates may vary over time and are particularly useful for comparing investments across time periods and performance between different asset classes with different taxation.
- Leveraging a portfolio, via borrowing or futures, can amplify the portfolio’s gains or losses.

Contents

Keywords

I. Introduction

Introduction

Time Value of Money

The principles governing equivalence relationships between cash flows with different dates. Money has time value in that individuals value a given amount of money more highly the earlier it is received.

Keywords

II. Interest Rates and Time Value of Money

Discount

To reduce the value of a future payment in allowance for how far away it is in time. To calculate the present value of some future amount. The amount by which an instrument is priced below its face (par) value.

Interest Rate | **Yield**

A rate of return that reflects the relationship between differently dated cash flows. A discount rate.

Opportunity Cost

The value that investors forgo by choosing a particular course of action. The value of something in its best alternative use.

Keywords

II. Interest Rates and Time Value of Money

Real Risk-Free Interest Rate

The single-period interest rate for a completely risk-free security if no inflation is expected.

Inflation Premium

An extra return that compensates investors for expected inflation.

Nominal Risk-Free Interest Rate

The sum of the risk-free rate and the inflation premium.

Keywords

II. Interest Rates and Time Value of Money

Default Risk Premium

An extra return that compensates investors for the possibility that the borrower will fail to make a promised payment at the contracted time and in the contracted amount

Liquidity Premium

An extra return that compensates investors for the risk of loss relative to an investment's fair value if the investment needs to be converted into cash quickly.

Maturity Premium

An extra return that compensates investors for the increased sensitivity of the market value of debt to a change in market interest rate as maturity is extended.

II. Interest Rates and Time Value of Money

Determinants of Interest Rates

Interest Rates → Required Rate of Return

Interest Rates → Discount Rate

Interest Rates → Opportunity Cost

$$\mathrm{r} = \text{Real RFR} + \text{Inflation Premium} + \text{Default Risk Premium} + \text{Liquidity Premium} + \text{Maturity Premium}$$

- Real Risk-Free Interest Rate.
- Inflation Premium.
- Nominal Risk-Free Rate (Government Bond Yield).
- Default Risk Premium.
- Liquidity Premium.
- Maturity Premium.

$$\text{Nominal RFR} = \text{Real RFR} + \text{Inflation Premium}$$

$$(1 + \text{Real RFR}) = \frac{(1 + \text{Nominal RFR})}{(1 + \text{Inflation rate})}$$

Inflation Premium vs. Inflation Rate

RFR = Risk-Free Rate.

II. Interest Rates and Time Value of Money

Determining Interest Rates

In the following table we have information on five debt securities. All these securities embrace a unique single payment at maturity. Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

Investment	Maturity (in years)	Liquidity	Default Risk	Interest Rate (%)
1	2	High	Low	2.0
2	2	Low	Low	2.5
3	7	Low	Low	r_3
4	8	High	Low	4.0
5	8	Low	High	6.5

a) Explain the difference between the interest rates offered by investment 1 and investment 2.

- Investment 2 incorporates a liquidity premium of 0.5% (compensation for the lower liquidity of investment 2).

b) Estimate the default risk premium affecting all securities.

- Investment 4 and 5 have the same maturity, but different liquidity and default risk. From the 2.5% difference, 2% comes from default risk, and 0.5% from liquidity risk.

c) Calculate upper and lower limits for the unknown interest rate for Investment 3 (r_3).

- Investment 3 and 2 have comparable liquidity risk and default risk, but higher maturity risk. ($2.5\% < r_3 < 4.5\%$).

III. Rates of Return

Introduction

Return
- Holding Period Return
- Arithmetic or Mean Return
- Geometric Mean Return
- Money-Weighted Rate of Return
- Time-Weighted Rate of Return
- Annualized Return

Keywords

III. Rates of Return

Holding Period Return = Total Return

Holding Period Return	Total Return	Accumulated Return

The return that an investor earns during a specified holding period.

III. Rates of Return

Holding Period Return = Total Return

Return → Income Payments / Capital Gain/Loss

Total Return

$$\text{Percent} = 7\%$$

$$\text{Fraction} = \frac{7}{100}$$

$$\text{Decimals} = 0.07$$

Single Period Returns

$$\text{HPR} = \frac{P_1 + I_1 - P_0}{P_0}$$

$$\text{Gross Return} = R = 1 + \text{HPR}$$

$$\text{Net Return} = r = \text{HPR}$$

- We assume the income is received at the end (ease of illustration).

Multiple Period Returns

$$(1 + \text{HPR}) = (1 + R_1)(1 + R_2)\cdots(1 + R_n)$$

III. Rates of Return

Holding Period Return = Total Return

Suppose you bought 10,000 shares of company A at €89.45, five months ago. Today you receive a dividend of €4.51 per share, and you sell your stocks for €92.48. Determine the HPR of this trade.

$$\mathrm{HPR} = \frac{P_1 + D_1 - P_0}{P_0} = \frac{92.48 + 4.51 - 89.45}{89.45} = 0.0843 = 8.43\%$$

Suppose you invested $100,000 in a stock market mutual fund and the annual returns of the previous three years have been, 15%, -4%, and 19%, respectively. Determine the HPR of this investment.

$$(1 + \mathrm{HPR}) = (1 + R_1)(1 + R_2)(1 + R_3)$$

$$(1 + \mathrm{HPR}) = (1 + 0.15)(1 - 0.04)(1 + 0.19)$$

$$\mathrm{HPR} = (1 + 0.15)(1 - 0.04)(1 + 0.19) - 1 = 0.3138 = 31.38\%$$

III. Rates of Return

Arithmetic Return vs. Geometric Return

Arithmetic Returns

$$\bar{R}_i = \frac{R_{i,1} + R_{i,2} + \cdots + R_{i,T}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$$

- Easy to compute and known statistical properties (standard deviation).
- It assumes the amount invested at the beginning of each period is the same.

Geometric Returns

$$(1 + \bar{G}_i)^T = (1 + R_{i,1})(1 + R_{i,2}) \cdots (1 + R_{i,T})$$

- More accurate representation of the portfolio growth than the arithmetic mean.

- T is the total number of periods.
- $R_{i,t}$ is the return of i at perid t.

III. Rates of Return

Arithmetic Return vs. Geometric Return

Suppose you bought 10,000 shares of a high-yield bond market mutual fund and in the previous six years, the fund achieved the following returns: 4%, -2%, 8%, 12%, 19%, and -1%.

a) Determine the annual return using an arithmetic mean.

$$R_{\text{Arithmetic}} = \frac{0.04 - 0.02 + 0.08 + 0.12 + 0.19 - 0.01}{6} = 0.0667 = 6.67\%$$

b) Determine the annual return using a geometric mean.

$$(1 + R_{\text{Geometric}})^6 = (1.04)(0.98)(1.08)(1.12)(1.19)(0.99)$$

$$R_{\text{Geometric}} = [(1.04)(0.98)(1.08)(1.12)(1.19)(0.99)]^{\frac{1}{6}} - 1 = 0.0642 = 6.42\%$$

c) Which of these calculations better represents the annual growth of your investment?

$R_{\text{Geometric}}$ represents the compounding growth

Keywords

IV. Money-Weighted and Time-Weighted Return

MWRR versus TWRR

Time-Weighted Rate of Return	TWRR

The compound rate of growth of one unit of currency invested in a portfolio during a stated measurement period. A measure of investment performance that is not sensitive to the timing and amount of withdrawals or additions to the portfolio.

Keywords

IV. Money-Weighted and Time-Weighted Return

MWRR versus TWRR

Internal Rate of Return	IRR

The discount rate that makes net present value equal to 0. The discount rate that makes the present value of an investment's costs (outflows) equal to the present value of the investments' benefits (inflows).

Money-Weighted Rate of Return	MWRR

The internal rate of return on a portfolio, taking into account of all cash flows.

IV. Money-Weighted and Time-Weighted Return

MWRR versus TWRR

MWRR = IRR

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

- T is the total number of periods.
- CF_t is the cash flow at perido t.

- Neither the arithmetic nor the geometric mean accounts for cash flows in a portfolio.
- Preferred return methodology for the investor.

TWRR = Geometric Mean

$$(1+TWRR)^T = (1+R_1)(1+R_2)\cdots(1+R_T)$$

- Not sensitive to the additions and withdrawals of funds.
- Preferred return methodology for the portfolio manager.

1) Price the portfolio at each significant addition or withdrawal.
2) HPR for each sub-period.
3) Link the HPRs to get an annual return.

   - HPR: n < 1 year.
   - Geometric Mean: n > 1 year.

MWRR = Money-Weighted Rate of Return
IRR = Internal Rate of Return
TWRR = Time-Weighted Rate of Return

IV. Money-Weighted and Time-Weighted Return

MWRR (IRR)

Suppose an investor allocated $300 in a mutual fund three years ago, then $890 two years ago, and the investor withdraws $275 one year ago. The annual returns of the previous three years have been, respectively: -30%, 25%, and 10%. Determine the Money-Weighted Rate of Return.

$-\$300$	$-\$890$	$+\$275$	$+\$1210$
0	1	2	3

Year	1	2	3
Balance from previous year	$0	$210	$1100
New Investment (cash inflows) at the beginning of the year	$300	$890	0
Net Balance at the beginning of the year	$300	$1100	$1100
Annual Return	-30%	25%	10%
Investment Gain/Loss	-$90	$275	$110
Withdrawal by the investor (cash outflow) at the end of the year	0	-$275	0
Balance at the end of the year	$210	$1100	$1210

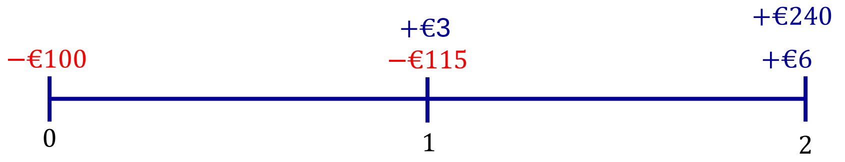
$$\frac{CF_0}{(1+IRR)^0}+\frac{CF_1}{(1+IRR)^1}+\frac{CF_2}{(1+IRR)^2}+\frac{CF_3}{(1+IRR)^3}=0$$

$$\frac{-300}{(1+IRR)^0}+\frac{-890}{(1+IRR)^1}+\frac{275}{(1+IRR)^2}+\frac{1210}{(1+IRR)^3}=0 \longrightarrow IRR=11.30\%$$

IV. Money-Weighted and Time-Weighted Return

MWRR (IRR)

Suppose an investor bought one share at time t=0 at a price of €100. At time t=1, the investor buys an additional share at €115. At the end of time t=2, the investor sells both shares for €120 each. During both years, the stocks pays a €3 dividend per share, and the investor does not reinvest dividends. Determine the Money-Weighted Rate of Return.



Time	Outflows
0	€100
1	€115

Time	Inflows
1	€3
2	€6
2	€240

$$\frac{\mathrm{CF}_0}{(1+\mathrm{IRR})^0}+\frac{\mathrm{CF}_1}{(1+\mathrm{IRR})^1}+\frac{\mathrm{CF}_2}{(1+\mathrm{IRR})^2}=0$$

$$\frac{-100}{(1+\mathrm{IRR})^0}+\frac{-112}{(1+\mathrm{IRR})^1}+\frac{246}{(1+\mathrm{IRR})^2}=0 \longrightarrow \mathrm{IRR}=10.5411\%$$

IV. Money-Weighted and Time-Weighted Return

MWRR (IRR)

Suppose you gather the following information of a mutual fund you are interested in:

Year	AUM at the beginning of the period	Net Return
1	$30,000,000	15%
2	$45,000,000	-5%
3	$20,000,000	10%
4	$25,000,000	15%
5	$35,000,000	3%

a) Compute the HPR for the five-year period.

$$(1 + \mathrm{HPR}) = (1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)(1 + R_5)$$

$$(1 + \mathrm{HPR}) = (1.15)(0.95)(1.1)(1.15)(1.03) = (1.4235) \longrightarrow \mathrm{HPR} = 42.34\%$$

b) Compute the arithmetic mean annual return.

$$\overline{R}_A = \frac{R_1 + R_2 + R_3 + R_4 + R_5}{T} = \frac{0.15 - 0.05 + 0.10 + 0.15 + 0.03}{5} = 0.076 = 7.60\%$$

c) Compute the geometric mean annual return.

$$(1 + \overline{R}_G)^T = (1 + R_1)(1 + R_2)\cdots(1 + R_T) \longrightarrow (1 + \overline{R}_G)^5 = (1.15)(0.95)\cdots(1.03) \longrightarrow \overline{R}_G = 7.32\%$$

d) Compare the arithmetic and the geometric mean annual returns.

$$\overline{R}_G \leq \overline{R}_A$$

AUM = Assets Under Management.

IV. Money-Weighted and Time-Weighted Return

MWRR (IRR)

Suppose you gather the following information of a mutual fund you are interested in:

e) Compute the MWRR for the five-year period.

Year	UAM at the beginning of the period	Net Return
1	$50,000,000	10%
2	$45,000,000	15%
3	$55,000,000	-5%
4	$60,000,000	20%
5	$70,000,000	5%

Year	1	2	3	4	5
Balance from previous year	$0	$55.00	$51.75	$52.25	$72.00
Withdrawal by the investor (cash outflow) at the beginning of the year	$00.00	-$10.00	$00.00	$00.00	-$02.00
New Investment (cash inflows) at the beginning of the year	$50.00	$00.00	$3.25	$07.75	$00.00
Net Balance at the beginning of the year	$50.00	$45.00	$55.00	$60.00	$70.00
Annual Return	10%	15%	-5%	20%	5%
Investment Gain/Loss	$05.00	$06.75	-$02.75	$12.00	$03.50
Balance at the end of the year	$55.00	$51.75	$52.25	$72.00	$73.50

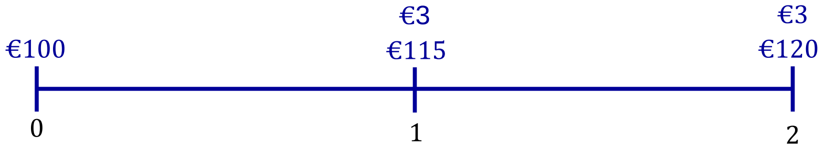
$$\frac{-50}{(1+\mathrm{IRR})^0}+\frac{+10}{(1+\mathrm{IRR})^1}+\frac{-3.25}{(1+\mathrm{IRR})^2}+\frac{-7.75}{(1+\mathrm{IRR})^3}+\frac{2}{(1+\mathrm{IRR})^4}+\frac{73.5}{(1+\mathrm{IRR})^5}=0$$

$$\mathrm{IRR}=8.83\%$$

IV. Money-Weighted and Time-Weighted Return

TWRR

Suppose an investor bought one share at time t=0 at a price of €100. At time t=1, the investor buys an additional share at €115. At the end of time t=2, the investor sells both shares for €120 each. During both years, the stocks pays a €3 dividend per share, and the investor does not reinvest dividends. Determine the Time-Weighted Rate of Return.



Time	Outflows
0	€100
1	€115

Time	Inflows
1	€3
2	€3
2	€120

$$\mathrm{HPR}_1 = \frac{115 + 3 - 100}{100} = 18\%$$

$$\mathrm{HPR}_2 = \frac{120 + 3 - 115}{115} = 6.96\%$$

$$(1 + \mathrm{TWRR})^2 = (1.18)(1.0696) \longrightarrow \mathrm{TWRR} = 12.34\%$$

IV. Money-Weighted and Time-Weighted Return

TWRR

Determine the TWRR for the following mutual fund you are interested in:

Quarter	1Q	2Q	3Q	4Q
Beginning Value	$2,000,000	$3,000,000	$3,250,000	$3,750,000
Inflow (outflow) at the beginning of the period	$500,000	($400,000)	$500,000	$1,250,000
Ending Value	$3,000,000	$3,250,000	$3,750,000	$3,500,000

$$\left\{\begin{aligned} HPR_{Q_1} &= \frac{\$3{,}000{,}000 - (\$2{,}000{,}000 + \$500{,}000)}{\$2{,}000{,}000 + \$500{,}000} = 20\% \\ HPR_{Q_2} &= \frac{\$3{,}250{,}000 - (\$3{,}000{,}000 - \$400{,}000)}{\$3{,}000{,}000 - \$400{,}000} = 25\% \\ HPR_{Q_3} &= \frac{\$3{,}750{,}000 - (\$3{,}250{,}000 + \$500{,}000)}{\$3{,}250{,}000 + \$500{,}000} = 0\% \\ HPR_{Q_4} &= \frac{\$3{,}500{,}000 - (\$3{,}750{,}000 + \$1{,}250{,}000)}{\$3{,}750{,}000 + \$1{,}250{,}000} = -30\% \end{aligned}\right.$$

$$(1 + TWRR) = (1.20)(1.25)(1.00)(0.70) = 1.05$$

$$TWRR = 5\%$$

IV. Money-Weighted and Time-Weighted Return

TWRR

Here we can see information about Mutual Fund Omega for 2022. It is a stock market mutual fund and all dividends received by portfolio companies are reinvested:

- 01/01/2022: Beginning portfolio value = $400 million
- 03/31/2022: Portfolio Value before Dividends = $430 million.
- 03/31/2022: Dividends received = $10 million.
- 06/30/2022: Portfolio Value before Dividends = $445 million.
- 06/30/2022: Dividends received = $5 million.
- 06/30/2022: New Investment = $300 million.
- 12/31/2022: Portfolio Value = $800 million.

a) Compute the TWRR.

$$\left\{\begin{array}{l} \mathrm{HPR}_1 = \dfrac{\$430 + \$10 - \$400}{\$400} = 10\% \\[2ex] \mathrm{HPR}_2 = \dfrac{\$445 + \$5 - \$430}{\$430} = 4.65\% \\[2ex] \mathrm{HPR}_3 = \dfrac{\$800 - (\$445 + \$300)}{\$445 + \$300} = 7.38\% \end{array}\right.$$

$$(1 + \mathrm{TWRR}) = (1.10)(1.0465)(1.0738) = 1.2361$$

$$\mathrm{TWRR} = 23.61\%$$

IV. Money-Weighted and Time-Weighted Return

TWRR

Here we can see information about Mutual Fund Omega for 2022. It is a stock market mutual fund and all dividends received by portfolio companies are reinvested:

- 01/01/2022: Beginning portfolio value = $400 million
- 03/31/2022: Portfolio Value before Dividends = $430 million.
- 03/31/2022: Dividends received = $10 million.
- 06/30/2022: Portfolio Value before Dividends = $445 million.
- 06/30/2022: Dividends received = $5 million.
- 06/30/2022: New Investment = $300 million.
- 12/31/2022: Portfolio Value = $800 million.

b) Compute the MWRR.

$$\frac{-400}{(1+\mathrm{IRR})^0}+\frac{0}{(1+\mathrm{IRR})^1}+\frac{-300}{(1+\mathrm{IRR})^2}+\frac{0}{(1+\mathrm{IRR})^3}+\frac{800}{(1+\mathrm{IRR})^4}=0$$

$$\mathrm{IRR}_{\mathrm{Quarterly}}=4.31\%$$

$$\left(1+\mathrm{IRR}_{\mathrm{Annually}}\right)=\left(1+\mathrm{IRR}_{\mathrm{Quarterly}}\right)^4$$

$$\mathrm{IRR}_{\mathrm{Annually}}=\left(1+\mathrm{IRR}_{\mathrm{Quarterly}}\right)^4-1=(1+0.0431)^4-1=18.39\%$$

IV. Money-Weighted and Time-Weighted Return

TWRR

Here we can see information about Mutual Fund Omega for 2022. It is a stock market mutual fund and all dividends received by portfolio companies are reinvested:

- 01/01/2022: Beginning portfolio value = $400 million
- 03/31/2022: Portfolio Value before Dividends = $430 million.
- 03/31/2022: Dividends received = $10 million.
- 06/30/2022: Portfolio Value before Dividends = $445 million.
- 06/30/2022: Dividends received = $5 million.
- 06/30/2022: New Investment = $300 million.
- 12/31/2022: Portfolio Value = $800 million.

c) Interpret the differences between the MWRR and the TWRR.

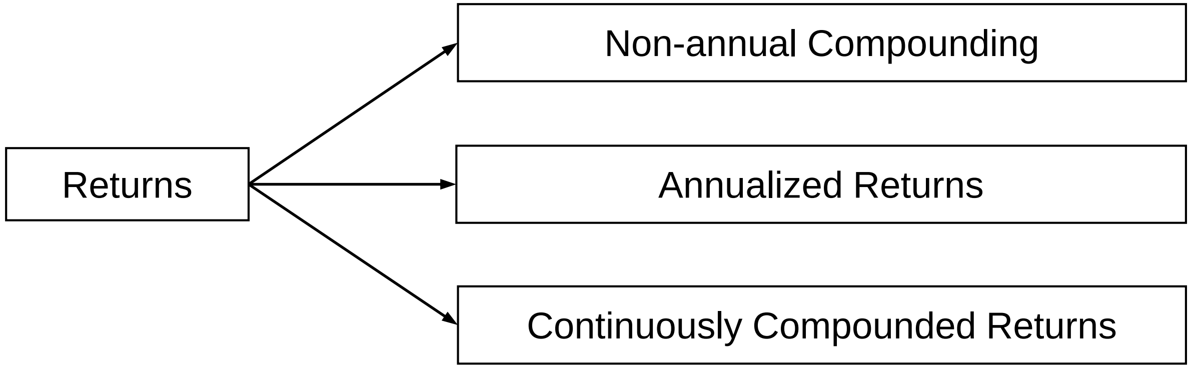
$$\text{TWRR} = 23.61\%$$

$$\text{MWRR} = 18.39\%$$

MWRR is lower than the TWRR because a large inflow entered in the last 6 months, while the larger return of this fund came in the first 6 months.

V. Annualized Return

Introduction



V. Annualized Return

Non-annual Compounding

$$\boxed{FV_n = PV\left(1 + \frac{R}{m}\right)^{m \cdot n}}$$

- m is the number of compounding periods per year.
- R is the quoted annual interest rate.
- n is the number of years.

V. Annualized Return

Non-annual Compounding

You are a pension fund manager and you have to make a lump-sum payment of 20 million euros, 20 years from today. How much money should you have right now if you can capitalize that amount monthly, at a 5% annual stated return?

$$FV_n = PV\left(1 + \frac{R}{m}\right)^{m \cdot n}$$

$$€20{,}000{,}000 = PV\left(1 + \frac{0.05}{12}\right)^{12 \cdot 20}$$

$$PV = €7{,}372{,}891$$

Keywords

V. Annualized Return

Continuously Compounded Return

Continuously Compounded Return

The natural logarithm of 1 plus the holding period return, or equivalently, the natural logarithm of the ending price over the beginning price.

$$r = \mathrm{Ln}(1 + \mathrm{HPR})$$

V. Annualized Return

Continuously Compounded Return

Continuously Compounded Return

$$r = \text{Ln}(1+R) = \text{Ln}\left(\frac{P_T}{P_0}\right)$$

Apply the exponential function

$$e^r = \frac{P_T}{P_0}$$

$$P_T = P_0 \cdot e^r$$

- r is the continuously compounded return.
- R is the Holding Period Return.
- P_0 is the initial price.
- P_T is the final price.

$$\frac{P_T}{P_0} = \frac{P_T}{P_{T-1}} \cdot \frac{P_{T-1}}{P_{T-2}} \cdots \frac{P_1}{P_0}$$

Apply the log function

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

V. Annualized Return

Continuously Compounded Return

Suppose you invested €1,000, 6 years ago and you are interested in knowing the holding period return, and the continuously compounded return, if you have €3,000 right now.

$$\text{HPR} = \frac{3{,}000 - 1{,}000}{1{,}000} = 2 = 200\%$$

$$\text{r} = \text{Ln}(1 + \text{HPR}) = \text{Ln}(1 + 2) = 1.09861 = 109.8\%$$

V. Annualized Return

$$\boxed{(1 + R_{\text{Annualized}})^{\text{years}} = (1 + \text{HPR})}$$

1 year = 365 days.
1 year = 52 weeks.
1 year = 12 months.

$\text{HPR}_{250\text{ days}} \longrightarrow (1 + R_{\text{Annualized}})^{\frac{250}{365}} = \left(1 + \text{HPR}_{250\text{ days}}\right)$

$\text{HPR}_{25\text{ weeks}} \longrightarrow (1 + R_{\text{Annualized}})^{\frac{25}{52}} = (1 + \text{HPR}_{25\text{ weeks}})$

$\text{HPR}_{42\text{ months}} \longrightarrow (1 + R_{\text{Annualized}})^{\frac{42}{12}} = (1 + \text{HPR}_{42\text{ months}})$

$\text{HPR}_{3\text{ years}} \longrightarrow (1 + R_{\text{Annualized}})^{3} = \left(1 + \text{HPR}_{3\text{ years}}\right)$

V. Annualized Return

An analyst is interested in making a comparison among four different investments so he decides to calculate the annualized return for all of them:

- Security A has earned a 3.2% in the previous 150 days.
- Security B has earned a 6.2% in the previous 30 weeks.
- Security C has earned a 7.8% in the previous 15 months.
- Security D has earned a 25.2% in the previous 4 years.

$$(1 + R_{\text{Annualized}})^{\text{years}} = (1 + \text{HPR})$$

$$(1 + R_A)^{\frac{150}{365}} = (1 + 0.032) \longrightarrow R_A = 7.97\%$$

$$(1 + R_B)^{\frac{30}{52}} = (1 + 0.062) \longrightarrow R_B = 10.99\%$$

$$(1 + R_C)^{\frac{15}{12}} = (1 + 0.078) \longrightarrow R_C = 6.19\%$$

$$(1 + R_D)^{4} = (1 + 0.252) \longrightarrow R_D = 5.74\%$$

VI. Other Major Return Measures and their Applications

Leverage

In the context of corporate finance (corporate issuers), leverage refers to the use of fixed costs within a company's cost structure. **Fixed costs** that are operating costs (such as depreciation or rent) create *operating leverage*. Fixed costs that are financial costs (such as interest expense) create *financial leverage*.

In the context of **investments**, it refers to the relationship between the equity and the nominal or effective amount.

VI. Other Major Return Measures and their Applications

Returns are influenced by → Fees, Taxes, Inflation, Leverage

1 Gross and Net Returns

- Net Return deducts managerial and administrative expenses.
- Small funds with a limited amount of AUM are at a disadvantage.

2 Pre-tax and after-tax Nominal Returns

- As a general rule, all returns are pre-tax nominal returns.
- Capital gains (short-term and long-term) versus income.
- Taxes can be minimized reducing trading turnover and selecting appropriate securities.

AUM = Assets Under Management

Keywords

VI. Other Major Return Measures and their Applications

Risk Premium

An extra return expected by investors for bearing some specified risk.

VI. Other Major Return Measures and their Applications

3 Nominal Returns versus Real Returns

$$(1 + r_N) = (1 + r_R)(1 + \pi)(1 + RP)$$

$$(1 + r_N) = (1 + r_R)(1 + \pi)$$

- r_N is the nominal rate of return.
- r_R is the real rate of return (compensation for postpoing consumption).
- π is the inflation rate (compensation for loss of purchasing power).
- RP is the Risk Premium (compensation for assuming risk).

4 Leveraged Return

- Take a derivative position (e.g. futures, options, etc.).
- Take a position borrowing money (e.g., stocks, real estate, etc.).

VI. Other Major Return Measures and their Applications

Suppose you invest in a security receiving an annual return of 11%. If your annual tax rate is 20%, calculate the after-tax return of your investment.

$$r_{\text{after tax}} = r_{\text{before tax}}(1 - 0.20) = 11(1 - 0.20) = 8.8\%$$

Suppose you invest in a security receiving an annual return of 11%. If your annual tax rate is 20%, and the annual inflation rate is 2%, calculate the after-tax real return of your investment.

$$r_{\text{after tax}} = r_{\text{before tax}}(1 - 0.20) = 11(1 - 0.20) = 8.8\%$$

$$(1 + r) = \frac{(1 + 0.088)}{(1 + 0.02)} = 1.06666$$

$$r = 6.67\%$$

VI. Other Major Return Measures and their Applications

Suppose you gather the following historic asset class geometric returns:

Asset Class	Geometric Returns (%)
Equities	10.0
Corporate Bonds	7.5
Treasury Bills	3.5
Inflation	2.0

a) Calculate the real rate of return for equities.

$$\frac{(1+0.10)}{(1+0.02)} - 1 = 0.0784 = 7.84\%$$

b) Calculate the risk premium for equities.

$$\frac{(1+0.10)}{(1+0.035)} - 1 = 0.0628 = 6.28\%$$

FinancerTraining.com